Sub-space models

In many cases, we can assume that signals are low-dimensional in the sense that a high-dimensional observation $y\in{\mathbb R}^{N\times 1}$ can be completely explained by a low-dimensional representation $x\in{\mathbb R}^{M\times 1}$ such that with a matrix $A\in{\mathbb R}^{N\times M}$ we have $y = Ax$ with $N>M$. This signal thus spans only a *M*-dimensional *sub-space* of the whole *N*-dimensional space.

Application with known sub-space

This representation comes in handy for example if we assume that we have only a noisy observation of *y*. The desired signal lies in a sub-space, so all the other dimensions have only noise in them and we can remove them. We thus only need a mapping from the whole space to the sub-space. It turns out that such a mapping is a projection to the sub-space spanned by matrix *A*. In fact, the minimum mean square error (see Linear regression) solution is exactly the Moore-Penrose pseudo-inverse. However, the downside with this approach is that here the matrix A needs to be known in advance such that the pseudo-inverse can be formed.

Estimation of unknown sub-space

In practical cases it is rather unusual that we would have access to the matrix *A*, but instead, it must be estimated from available data. A typical approach is based on a singular value decomposition (SVD) or the eigen value decomposition. In short, we first estimate the covariance matrix of the signal and then decompose it into uncorrelated components with the singular value decomposition. Often, a small set of singular values make up most of the energy of the whole signal. Thus if we discard the smallest singular values, we do not loose much of the energy, but have a signal of a much lower dimensionality. The singular value decomposition thus takes the role of the sub-space mapping matrix *A*, and we can apply the model as described above.

Discussion

Sub-space models are theoretically appealing models, since their analysis is straightforward. In terms of speech signals, the difficulty lies in finding a representation which is actually low-rank. In other words, it is not immediately clear in which domain we can apply analysis such that speech signals can efficiently modelled by low-rank models.