

# Private Membership Test Protocol with Low Communication Complexity

➤ **Introduction:** Private membership test (PMT) protocols enable clients to query for a certain item in server's database without revealing to the server what the item is. Clients are also prevented from learning anything else about the server's database.

➤ **Protocol:** We present our protocol in a realistic setting where server possesses  $2^{21}$  malware samples and stores their hash values in 16 different subsets, based on the first 4 bits of the hash value. Each subset has  $2^{17}$  items. Client wants to privately search for an item with hash value equal to  $h$ .

## Server

- Divides each subset into 64 segments based on the next 6 bits of the hash value (bits 5,...,10). Each segment has  $2^{11}$  items.
- Inserts each segment into a Bloom filter with 10 hash functions. Each filter has  $2^{15}$  bits.
- Divides each filter into 16 parts and arranges sixteen  $2^3 \times 2^3$  matrices  $M$ , with parts of the filters as their elements, in such a way that the concatenation of all 16 elements in a particular position of all matrices, results to one of the Bloom filters.

## Client

- Generates two distinct  $2^{10}$ -bit primes  $p$  and  $q$  for Paillier cryptosystem.  $N = pq$  thus  $N$  has  $2^{11}$  bits.
- Calculates the integer corresponding to 5th, 6th and 7th bits of  $h$  as  $i^*$  and the integer corresponding to 8th, 9th and 10th bits as  $j^*$ .
- Arranges an 8-component vector  $\hat{e}_{i^*}$  (resp.  $\hat{e}_{j^*}$ ), where all the components are 0 except the component  $i^*$  (resp.  $j^*$ ), that is equal to 1.
- Encrypts the components of  $\hat{e}_{i^*}$  (resp.  $\hat{e}_{j^*}$ ) and generates vector  $(\alpha_i)$  (resp.  $(\beta_j)$ ).

$(\alpha_i), (\beta_j), N$  and first 4 bits of  $h$

For each matrix  $M$ :

- Computes  $\sigma_i = \prod_{j=0}^7 \beta_j^{M(i,j)} \bmod N^2 = E(M(i, j^*))$ .
- Represents  $\sigma_i = u_i N + v_i; i = 0, 1, \dots, 7$ .
- Computes  $u = \prod_{i=0}^7 \alpha_i^{u_i} \bmod N^2 = E(u_{i^*})$ .
- Computes  $v = \prod_{i=0}^7 \alpha_i^{v_i} \bmod N^2 = E(v_{i^*})$ .

- ❖ Client spends 1.8s to generate  $(\alpha_i)$  and  $(\beta_j)$ .
- ❖ Client sends 8 KB data to the Server.

$(u, v)$  for each matrix  $M$

- ❖ Server spends 0.9s to generate  $u_i$  and  $v_i$ .
- ❖ Server sends 16 KB data to the Client.
- ❖ Bloom filter has a false positive rate of 0.001.

- Decrypts all the sixteen pairs:  
 $D(D(u)N + D(v)) = D(u_{i^*}N + v_{i^*}) = D(\sigma_{i^*}) = M(i^*, j^*)$ .
- Concatenates the results into a Bloom filter.
- Performs membership test without the Server.

➤ **Conclusion:** Homomorphic encryption allows server to search in the matrix without knowledge of client's private key. A reasonable compromise in privacy allows significantly smaller communication complexity than prior art.

- ✓ Paillier, Pascal. "Public-key cryptosystems based on composite degree residuosity classes." In *International Conference on the Theory and Applications of Cryptographic Techniques*, pp. 223-238. Springer Berlin Heidelberg, 1999.
- ✓ Chang, Yan-Cheng. "Single database private information retrieval with logarithmic communication." In *Australasian Conference on Information Security and Privacy*, pp. 50-61. Springer Berlin Heidelberg, 2004.